Midterm Mathematical Physics

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- **Total number of 100 points**
- **10 points free for coming to the midterm exam**
- **Justify your answers for all problems**
- **Good luck!**

Problem 1

Determine the value of c for which

$$\sum_{n=0}^{\infty} e^{nc} = D\,(>1)$$

15 pts

Problem 2

(a) Compute the following limit

$$\lim_{n\to\infty} n^{3.5} \sin\left(\frac{1}{n^{3.5}}\right)$$

10 pts

(b) Determine whetether the following series is convergent

$$\sum_{n=1}^{\infty} n^{3.5} \sin\left(\frac{1}{n^{3.5}}\right)$$

5 pts

(c) Determine whether the following series is convergent

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^{3.5}}\right)$$

10 pts

Problem 3

Consider the following series

$$\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3} \qquad x \in \mathbb{R}$$

(a) For which values of x does this series converge?

15 pts

(b) For which values of x does this series diverge?

5 pts

Problem 4

Suppose a mass m is attached to a spring with spring constant k and let $k = m\omega^2$.

If an external force $F(t) = F_0 \cos(\omega t)$ is applied, then the equation of motion for non-zero dissipation ($c > 0$) has the form

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

If we assume $c^2 - 4mk < 0$, then using the method of undetermined coefficients show that the motion of the mass is given by

$$x(t) = e^{\frac{-ct}{2m}}(c_1 \cos(\widetilde{\omega} t) + c_2 \sin(\widetilde{\omega} t)) + \frac{F_0}{c\omega}\sin(\omega t)$$

where $\widetilde{\omega} = \omega\sqrt{1 - (\frac{c}{2m\omega})^2}$

30 pts